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AN ANALYTICAL AND EXPERIMENTAL APPROACH TO THE PENETRATION OF SEMI-INFINITE TARGETS BY LONG RODS

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ABSTRACT

The one-dimensional analysis of long rod penetration into semi-infinite targets as developed by Tate has previously been modified by the present authors to include the effects of mass transfer into the plastic zone and the mushroom strain at the penetrator tip. This latter factor has a substantial effect on calculated penetrations. This paper reviews the modified theory, extends its application to several cases not previously treated, and then details the results of a series of experiments that tend to confirm the validity of the modified mathematical model.

NOTATION

A	initial cross-sectional area of rod
c	$[Y/\rho]^{1/2}$
e	engineering strain
F	internal force
L	initial length of rod
ℓ	remaining undeformed length of rod
ℓ_f	final remaining undeformed length of rod
P	interface force
p	interface pressure
R	target strength
t	elapsed time since impact
u	speed of penetration into target
V	initial impact speed of rod
v	current speed of rod
X	length of rod that has been consumed
Y	rod strength
z	penetration depth
z_f	final penetration depth
Δ	increment
λ^2	ratio of target strength to that of rod
μ^2	ratio of mass density of target to that of rod
ρ	mass density of rod
$\cdot$	derivative with respect to time

INTRODUCTION

A problem of obvious technological significance is the penetration of armor by projectiles. This problem has received considerable attention over the last few decades. At one stage, the emphasis was on improved design of body armor; at another, it was enhanced penetrator design. During this same period the development of very fast, high-capacity computers enabled the penetration problem to be modeled with great detail, provided that the target and projectile materials were adequately described. However, the corresponding descriptions of the penetration process were far more detailed than any experimental results with which they might be compared.

From the experimental point of view, the mechanics of penetration has been studied for many years. However, slow progress with analytical techniques has caused most of the efforts to be supported by numerical studies. Nevertheless, simple one-dimensional theories have held the greatest attraction for most experimentalists. Among such theories, that of Tate [1,2] seems to have attracted the greatest following.

Whatever approach is adopted, and whatever theories employed, the design engineer confronted with high speed penetration faces a difficult problem. Materials selection under very high rates of loading has confronted and confounded designers for many decades. A thorough understanding of all the relevant material parameters and their effects on flow and fracture would be required for the selection of materials which are optimally suited for particular high strain rate applications. However, since this information is usually not available, other approaches are required. This is the motivation behind the use of one-dimensional engineering models. Even though such an approach is generally not capable of providing detailed information on specific aspects of the events, it can provide valuable insights into the interactions of the parameters in the problem.

Recently, the one-dimensional long rod penetration theory of Tate [1,2] was modified by the present authors [3,4] so as to account for mass loss from the undeformed section through the rigid-plastic interface. This modification makes the equation of motion exact instead of approximate. Mathematically, it adds a relative motion term to the existing equation of motion. This presented a slight increase in complexity, but introduced a more realistic, non-zero strain at the penetrator tip. Being able to take account of the tip strain of such a mushrooming penetrator is a substantial improvement over the original theory. Calculations based on this modification yield quite reasonable estimates of penetration depths using realistic values of the target and penetrator strengths. In the present paper, we review some of the details of the extension of Tate's theory. Then, several cases are treated in which solutions can be obtained in closed form. After that, some recent experimental results are presented that tend to confirm the validity of this modified theory.

THEORY

Consider the normal penetration of a semi-infinite target by an axisymmetric, cylindrical rod of initial length L. The rod material can be separated into rigid and plastic regions during the penetration process (Fig. 1). The plastic region is wafer thin and instantaneously eroding. However, the mean engineering strain in the plastic region (mushroom) is non-zero. This allows for the deformed and undeformed sections of the rod to have different diameters.

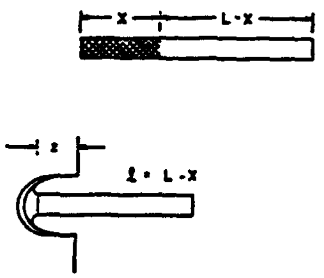

Fig. 1 Schematic of rod. (a) Shows plastic portion X, and undeformed portion L-X=ℓ. (b) Shows penetration into target to a depth z.

In [3], the authors examined the balance of impulse and momentum across the deformed and undeformed sections (Fig. 2). Application of this principle leads to the equation of motion for the undeformed section.

$$\ell\dot{v} + \dot{\ell}\,(v-u) = -p/\rho\,(1+e) \tag{1}$$

In this equation, v is the velocity of the undeformed section, u is the penetration velocity, ℓ is the undeformed section length, ρ is the constant penetrator mass density, p is the interface pressure between the penetrator tip and the target, and e is the engineering strain in the deformed section of the penetrator adjacent to the rigid-plastic interface. Dots over characters denotes time differentiation.

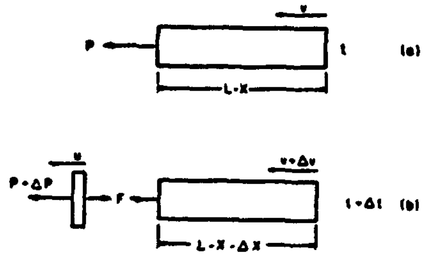

Fig. 2 Schematic of the transfer of mass element $\rho A\Delta X$ from the undeformed to the plastic portion of rod.

The velocities u and v are coupled by a kinematical relationship. This relationship can be derived by equating the distances in Figure 3. At time t, the undeformed section has length ℓ and speed v. At a later time, t + Δt, the undeformed section has lost an increment of length $\Delta\ell$ to plastic deformation. *Since the engineering strain at this point is e*, the corresponding length of the newly deformed section (mushroom) is $\Delta\ell(1 + e)$. The total distance ℓ + u Δt must equal v Δt + (ℓ - $\Delta\ell$) + $\Delta\ell$ (1 + e). Therefore, u Δt = v Δt + e $\Delta\ell$. Dividing by Δt and passing to the limit as Δt → 0, leads to

$$e\dot{\ell} = v - u \tag{2}$$

Notice that Fig. 3 requires $-\Delta\ell/\Delta t$ to go to $\dot{\ell}$. This kinematical equation replaces the classical equation $\dot{\ell} = -(v-u)$ used by Tate [1,2].

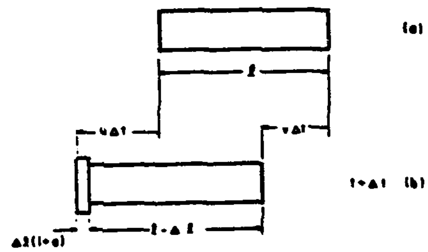

Fig. 3 Schematic illustration of the rear portion of the projectile. During the time interval Δt, the front of the indicated section is displaced a distance uΔt, while the rear is displaced a distance vΔt.

Tate [1,2] has advocated the use of a modified Bernoulli equation to complete the system (see, also Tate [5]). This equation gives both an estimate of the interface pressure p and a relationship between u and v

$$p = \frac{1}{2}\mu^2\rho\, u^2 + R = \frac{1}{2}\rho(v-u)^2 + Y \tag{3}$$

where $\mu^2\rho$ denotes the target density and R and Y are the dynamic strength factors of the target and penetrator respectively. In general, these factors are quite different from the static yield strengths for the materials.

Equations (1)-(3) form a system of three first order ordinary differential equations in four time-dependent unknowns: ℓ, v, u, and e. The modified Bernoulli equation stems from pseudo-steady-state considerations. It is appropriate, therefore, to take e to be constant. The target craters are approximated by cylindrical holes of depth $\bar{z}$ and cross-sectional area $A/(1 + e)$, where A is the initial cross-sectional area of the rod. For the purpose of comparing the theory with experiment, the values of e will be computed from the "profile hole diameters" of the targets. The "profile hole diameter" is the minimum diameter of the penetration crater produced in the target (see Fig. 4).

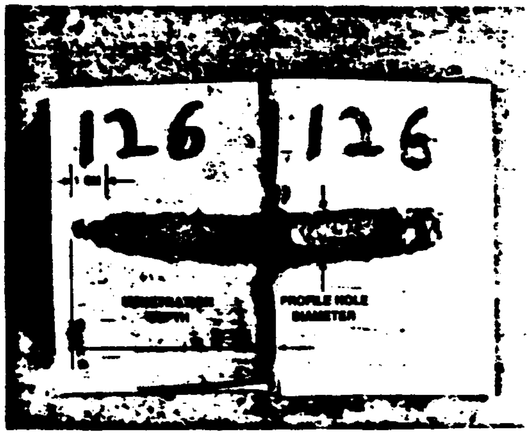

Fig. 4 Portion of a 7075-T6 aluminum target after being impacted at 2.49 km s^{-1} by a hard steel rod. The target has been sectioned through the resulting crater to show its shape.

By taking e as an experimentally determined parameter, equations (1)-(3) represent a system of three first order ordinary differential equations in the time-dependent unknowns ℓ, v, u. When data for material properties is supplied, these equations can be integrated and the total penetration depth z_f can be computed from

$$\dot{z} = u \tag{4}$$

The initial conditions for the system are $v(0) = V$ and $\ell(0) = L$, where V is the impact velocity of the rod. The initial penetration velocity u(0) can be found from equation (3).

INTEGRATION OF THE DIFFERENTIAL EQUATIONS

For the most part, analytical solutions to the system (1)-(3) are not available. There are some cases, however, where exact integrals are available. They are: (a) when the rod disintegrates at the target surface - no penetration; (b) when the rod penetrates without deforming - rigid body penetration; and (c) when the rod and target are of equal strengths. These cases were originally treated by Tate [1,2] and later by the present authors [3] for Tate's kinematical relation $\dot{\ell} = -(v-u)$, but not for the more general kinematical relation presented in this paper, $\dot{\ell} = (v-u)/e$, equation (2). These important cases will be treated in the order in which they were presented in [3].

(a) <u>No Penetration</u>

No penetration is implied by $u = 0$. The pressure at the rod/target interface can be formed from (3).

$$p = \frac{1}{2}\rho v^2 + Y \tag{5}$$

From (2), $v = e\dot{\ell}$ and $\dot{v} = e\ddot{\ell}$ which can be substituted into (1) to obtain

$$\ell \ddot{\ell} + \frac{(2 + 3e)}{2(1+e)} \dot{\ell}^2 + \frac{Y}{\rho e(1+e)} = 0 \tag{6}$$

This equation can be integrated once, subject to the initial conditions $\ell(0) = L$ and $\dot{\ell}(0) = V/e$. The result is

$$\dot{\ell}^2 = \left[\frac{V^2}{e^2} + \frac{2Y}{\rho e(2+3e)}\right]\left(\frac{L}{\ell}\right)^{\frac{2+3e}{1+e}} - \frac{2Y}{\rho e(2+3e)} \tag{7}$$

When the event is over $v = 0$, which means that $\dot{\ell} = 0$, $\ell = \ell_f$, and

$$\left(\frac{L}{\ell_f}\right)^{-\frac{2+3e}{1+e}} = 1 + \frac{2+3e}{2e}\left(\frac{V}{c}\right)^2 \tag{8}$$

where $c^2 = Y/\rho$ and ℓ_f is the length of rod remaining after it comes to rest. The rod is entirely consumed when

$$\frac{2+3e}{2e}\left(\frac{V}{c}\right)^2 = -1 \tag{9}$$

The corresponding strain satisfies

$$\left[2 + 3\left(\frac{V}{c}\right)^2\right] e + 2\left(\frac{V}{c}\right)^2 = 0 \tag{10}$$

The strain given by equation (10) is the minimum value of e that can be sustained during this impact.

When e passes from compressive strains which are less than 2/3 to compressive strains which exceed 2/3 the exponent of the ratio L/ℓ_f changes sign. However, equation (8) continues to be valid, although the algebraic structure is different.

(b) <u>Rigid Body Penetration</u>

In this case, there is no rod deformation so that $u = v$ and $\dot{\ell} = 0$. The pressure at the rod/target interface is found from (3).

$$p = \frac{1}{2}\mu^2\rho u^2 + R \tag{11}$$

Since $\dot{\ell} = 0$, $\ell = L$ throughout the event and equation (1) becomes

$$\dot{u} + \frac{\mu^2}{2L(1+e)} u^2 + \frac{R}{\rho L(1+e)} = 0 \qquad (12)$$

Now, using this result, we can compute the penetration depth. From (4) $\dot{z} = u$ which can be used to transform $\dot{u} = u\, du/dz$. Thus, (12) becomes

$$dz = \frac{-u\, du}{\frac{\mu^2}{L(1+e)} u^2 + \frac{R}{\rho L(1+e)}} \qquad (13)$$

which can be directly integrated to give

$$z = \frac{1+e}{\mu^2} L \ln \left[\frac{V^2 + \frac{2R}{\rho\mu^2}}{u^2 + \frac{2R}{\rho\mu^2}} \right] \qquad (14)$$

In this equation, we have assumed that $z = 0$ at impact when $u = V$. The total penetration depth z_f occurs when the penetration velocity u reaches zero.

$$z_f = \frac{1+e}{\mu^2} L \ln \left[1 + \frac{\rho\mu^2 V^2}{2R} \right] \qquad (15)$$

This formula is similar to the classical De Marre formula (see Reinhart and Pearson [6]) or the Petry equation (see Backman [7]). It is also the same as that previously presented by the authors [3].

(c) Equal Strengths

As indicated earlier, the penetration-with-consumption case is not integrable, in general. However, the case for equal target and penetrator strengths, $R = Y$, is an isolated example of rod penetration, accompanied by deformation and erosion, which is integrable.

The condition of equal strengths reduces (3) to

$$u = \frac{1}{1 + \mu} v \qquad (16)$$

With this result, (2) becomes

$$e\dot{\ell} = \frac{\mu}{1 + \mu} v \qquad (17)$$

Using equations (16) and (17), we can substitute into (1) to obtain a single equation for ℓ.

$$\ell \ddot{\ell} + a \dot{\ell}^2 - b = 0 \qquad (18)$$

where

$$a = \frac{\mu(2+3e)}{2(1+\mu)(1+e)} \qquad (19)$$

and

$$b = \frac{-\mu R}{\rho e(1+e)(1+\mu)} \qquad (20)$$

The integration of (18) now proceeds by considering two separate cases.

Case (i) $a \neq 0$ or all possible compressive (engineering) strains except $e = -2/3$.

In this case, a first integral of (18) is

$$\dot{\ell}^2 + \frac{2R}{\rho e(2+3e)} = \left[\frac{\mu^2 V^2}{e^2(1+\mu)^2} + \frac{2R}{\rho e(2+3e)} \right] \left(\frac{L}{\ell}\right)^{2a} \qquad (21)$$

where the initial condtions $\ell(0) = L$ and $\dot{\ell}(0) = \mu V/(1+\mu)e$ have been employed. At the end of the event, $\dot{\ell} = 0$ and $\ell = \ell_f$, where ℓ_f is the final unconsumed rod length. Equation (21) becomes

$$\left[\frac{\mu^2 V^2}{e(1+\mu)^2} + \frac{2R}{\rho(2+3e)} \right] \left(\frac{L}{\ell_f}\right)^{2a} = \frac{2R}{\rho(2+3e)} \qquad (22)$$

This equation can be used to find ℓ_f in terms of the problem parameters. Whenever

$$\frac{\mu^2 V^2}{e(1+\mu)^2} + \frac{2R}{\rho(2+3e)} > 0 \qquad (23)$$

equation (22) can readily be solved for ℓ_f. For very small strains, this inequality will break down. Fortunately, these strains are too small to be realistic for most of the events that concern us. For very large strains, say $-1 < e < -2/3$, the signs on key terms in (22) change, including the sign of a. Thus, we can again find ℓ_f, even though the inequality in (23) is not satisfied.

The total penetration depth z_f is of considerable interest. This quantity can be found by noting that $u = e\dot{\ell}/\mu$ and $\dot{z} = u$ by equation (4). Eliminating u and integrating results in

$$z_f = \frac{-e}{\mu} (L - \ell_f) \qquad (24)$$

where ℓ_f is the solution of (22).

Case (ii) $a = 0$ or $e = -2/3$.

For the exceptional case, a first integral of (18) is

$$\dot{\ell}^2 = \frac{-2\mu R}{\rho e(1+e)(1+\mu)} \ln \left(\frac{\ell}{L}\right) + \frac{\mu^2 V^2}{e^2(1+\mu)^2} \qquad (25)$$

When the event concludes, there will be a final unconsumed rod length ℓ_f. This quantity is given by

$$\ell_f = L \exp \left\{ \frac{\rho\mu V^2(1+e)}{2eR(1+\mu)} \right\} \qquad (26)$$

The depth of penetration can be developed by observing that $u = -2\dot{\ell}/3\mu$. The result is

$$z_f = \frac{2}{3\mu} (L - \ell_f) \qquad (27)$$

where ℓ_f is taken from (26).

(d) General Case

For the general case of penetration accompanied by rod deformation when the target and rod have unequal strengths, numerical integration is required, as noted previously.

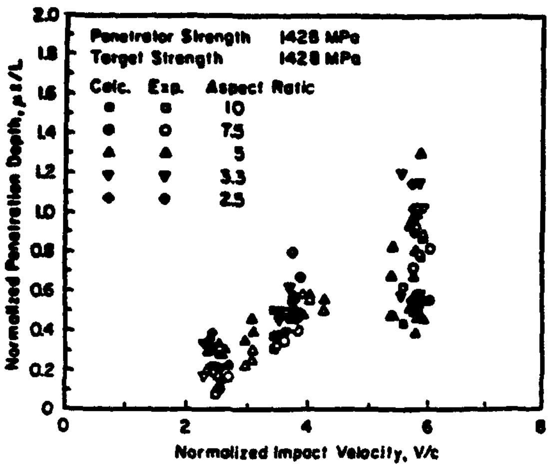

Fig. 5 Non-dimensionalized penetration depth versus non-dimensional impact velocity for hard steel targets by hard steel penetrators having various length-to-diameter ratios.

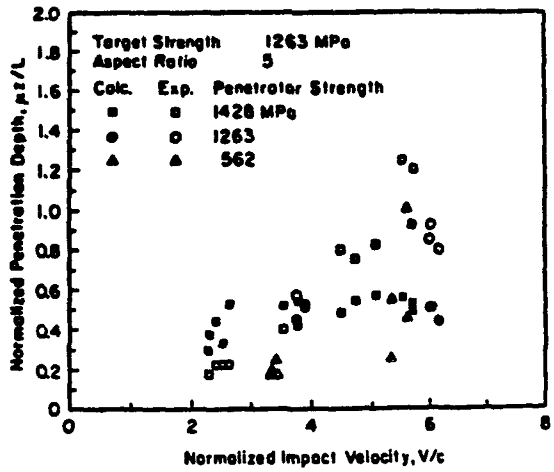

Fig. 6 Non-dimensionalized penetration depth versus non-dimensional impact velocity for soft steel targets and three different sets of penetrators: hard steel, soft steel and 7075 aluminum.

EXPERIMENTAL RESULTS

Details of the penetration experiments have been published elsewhere [4]. Briefly summarized they involved targets and penetrators made from two steels and two aluminums. Aspect ratios (length/diameter) of the rods ranged from 2.5 to 10 and impact velocities were approximately in the range 1-2.5 km/s.

Figure 5 shows actual experimental results (open symbols) and calculated values (solid symbols) from equations (22) and (24), for the penetration of hard steel targets by hard steel rods. The dynamic strength of the material is 1428 MPa, as estimated from Taylor Tests on the actual specimen material. The values of e have been estimated from post-test measurements on the profile hole diameters of sectioned targets (see Fig. 4). The comparison with experiment is generally favorable. As expected, better agreement is achieved for the longer rods (higher aspect ratios).

Results obtained by numerical integration of equations (1)-(4) are compared with actual experimental results in Figures 6-9. The targets are soft steel (1263 MPa), hard steel (1428 MPa), 2024 aluminum (395 MPa), and 7075 aluminum (562 MPa). Various penetrators are used. The penetrator strengths are indicated in the figures. All of the rods have an aspect ratio of 5.

The agreement of theory with experiment is generally good. It is especially so when one considers that the dynamic strengths are estimated from actual laboratory measurements and not taken as adjustable parameters. The authors' theory [8] was used to make the dynamic strength estimates.

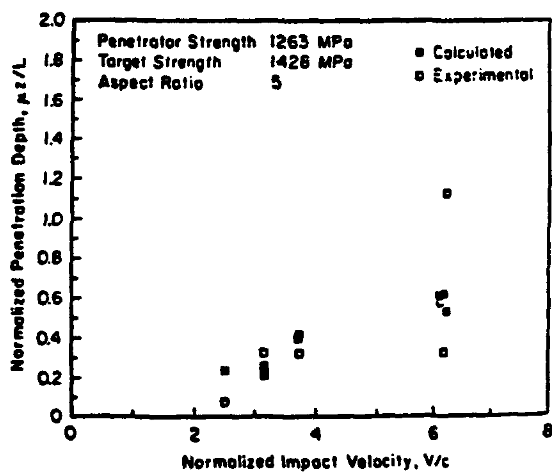

Fig. 7 Non-dimensionalized penetration depth versus non-dimensional impact velocity for hardened steel targets and soft steel penetrators.

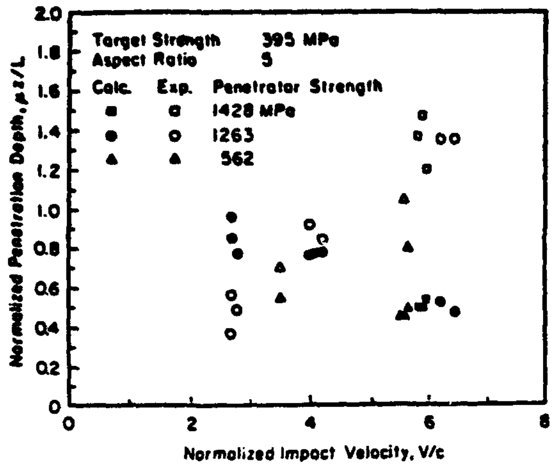

Fig. 8 Non-dimensionalized penetration depth versus non-dimensional impact velocity for 2024 aluminum targets and three different sets of penetrators: hard steel, soft steel and 7075 aluminum.

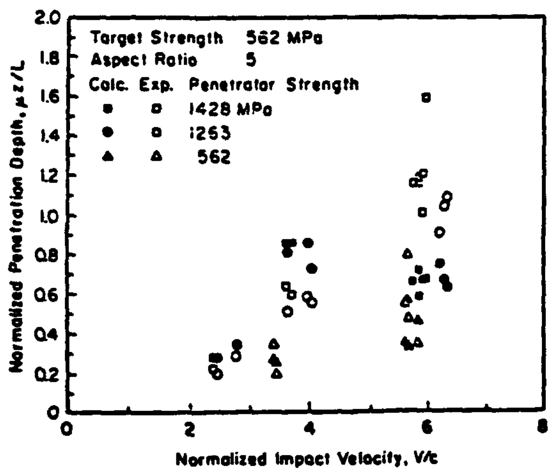

Fig. 9 Non-dimensionalized penetration depth versus non-dimensional impact velocity for 7075 aluminum targets and three different sets of penetrators: hard steel, soft steel and 7075 aluminum.

CONCLUSION

The theory of the mushrooming penetrator is a rather dramatic departure from the Tate Theory. Typically, the penetration depths predicted by the Tate Theory are about twice those predicted by the modified theory for the same material constants. The difference is primarily due to the lack of a mushroom on the tip of the penetrator in the earlier theory.

During the process of penetration, the eroding rod declerates as it goes into the target. It is quite likely that the mushroom strain at the rod end varies as the penetration velocity changes. This speculation is sustained by many experimental observations of the type shown by Fig. 4 in which the crater diameter clearly varies from end to end. On the other hand, this strain is simply taken as a constant in the present analysis. It is remarkable how such an approach can yield such good agreement with the experimental data.

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